

Worked answer key / 005

What Happens When You Run the Machine Again?

Each answer explains the reasoning as well as the result. The numbered prompts match the lesson and card deck.

005-1 / RECOGNIZE

Suppose $Av = 3v$ for a nonzero vector v . Read A^4v aloud, then state its value as a multiple of v .

$$A^4v = 3^4v = 81v.$$

1. The equation $Av = 3v$ verifies that v is an eigenvector with eigenvalue 3.
2. Each application multiplies this same vector direction by 3; four applications use four factors of 3.
3. Read A^4v as 'apply A four times to v ,' so the result is $81v$.

005-2 / RECOGNIZE

A nonzero eigenvector v has $\lambda = -2$. List its positions at steps $k = 0, 1, 2, 3$ as multiples of v . Does its length grow or shrink with each application, and does it point to the same side of the origin?

Its length doubles each step and its direction alternates: $v, -2v, 4v, -8v$.

1. The multipliers at steps 0 through 3 are 1, -2, 4, and -8.
2. Their absolute values are 1, 2, 4, and 8, so the arrow grows.
3. The signs alternate, so the arrow switches sides of the origin every step.

005-3 / APPLY

A nonzero eigenvector v has $\lambda = -1/2$. Write its positions at steps $k = 0, 1, 2, 3$, and 4 as multiples of v . Does any of these steps land on zero?

$v, -v/2, v/4, -v/8, v/16$. None is zero.

1. Begin with coefficient 1 at $k = 0$.
2. Successive multiplication by $-1/2$ gives $-1/2, 1/4, -1/8$, and $1/16$.
3. Every listed coefficient is nonzero. Indeed, $(-1/2)^k$ remains nonzero for every finite whole-number k .

Worked answers / continued

005-4 / APPLY

Let $A = \begin{bmatrix} 3 & 0 \\ 0 & -1 \end{bmatrix}$ and $v = (0, 2)$. Verify $Av = \lambda v$, identify λ , and find A^2v .

$Av = (0, -2) = -v$, so $\lambda = -1$ and $A^2v = v = (0, 2)$.

1. Matrix-vector multiplication gives $Av = (3 \cdot 0 + 0 \cdot 2, 0 \cdot 0 + (-1) \cdot 2) = (0, -2)$.
2. Since $v = (0, 2)$, the output is -1 times v .
3. The second application uses $(-1)^2 = 1$, so $A^2v = (0, 2)$.

005-5 / TRANSFER

Let $A = \begin{bmatrix} 2 & 0 \\ 0 & 1/2 \end{bmatrix}$ and $w = (1, 1)$. Compute Aw and A^2w . Why can you not choose one λ and write $A^2w = \lambda^2w$ here?

$Aw = (2, 1/2)$ and $A^2w = (4, 1/4)$. The first coordinate doubles while the second halves, so w is not an eigenvector and has no single λ for this rule.

1. One application gives $Aw = (2 \cdot 1, (1/2) \cdot 1) = (2, 1/2)$.
2. A second application gives $A(Aw) = (2 \cdot 2, (1/2) \cdot (1/2)) = (4, 1/4)$.
3. If Aw were λw , both coordinates would equal the same λ . They are 2 and $1/2$, so the eigenvector shortcut does not apply to w .

005-6 / TRANSFER

For $M = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$, compare the true matrix square M^2 with the array obtained by squaring each entry. Are they equal?

$M^2 = MM = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$, while entry-by-entry squaring gives $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$. They are different.

1. For MM , the top-right entry is $1 \cdot 1 + 1 \cdot 1 = 2$; the other entries are 1, 0, and 1.
2. Squaring the four entries separately gives 1, 1, 0, and 1.
3. A matrix power represents composition through matrix multiplication, which combines rows and columns.