

Worked answer key / 004

The Directions a Transformation Doesn't Turn

Each answer explains the reasoning as well as the result. The numbered prompts match the lesson and card deck.

004-1 / RECOGNIZE

A square matrix gives $Av = v$ for a nonzero v . Is v an eigenvector? What is λ ?

Yes. $\lambda = 1$.

1. Because v is nonzero and A is square, the definition can apply.
2. $Av = v = 1v$, so one common scalar works for every coordinate. The input is unchanged, not merely kept on the same line.

004-2 / RECOGNIZE

For $A = \begin{bmatrix} 3 & 0 \\ 0 & 2 \end{bmatrix}$, someone says $(1, 1)$ is an eigenvector because $A(1, 1) = (3, 2)$ still points generally up and right. Check the claim.

No. $(1, 1)$ is not an eigenvector of this A .

1. If $(3, 2) = \lambda(1, 1)$, the first coordinate demands $\lambda = 3$ while the second demands $\lambda = 2$.
2. One scalar cannot be both. A nearby-looking direction does not satisfy $Av = \lambda v$.

004-3 / APPLY

Keep $A = \begin{bmatrix} 3 & 0 \\ 0 & 2 \end{bmatrix}$. Test $v = (0, -3)$. Give Av and the eigenvalue if v qualifies.

$Av = (0, -6) = 2v$, so v is an eigenvector with $\lambda = 2$.

1. The first output coordinate is $3 \times 0 + 0 \times (-3) = 0$.
2. The second is $0 \times 0 + 2 \times (-3) = -6$. Because $(0, -6) = 2(0, -3)$, one common scalar works.

004-4 / APPLY

Let $D = \begin{bmatrix} -2 & 0 \\ 0 & 1 \end{bmatrix}$ and $v = (3, 0)$. Find Dv , decide whether v is an eigenvector, and describe what the sign of λ does.

$Dv = (-6, 0) = -2v$. Yes: $\lambda = -2$, so the arrow reverses along the same line and doubles in length.

1. The first coordinate is $(-2) \times 3 + 0 \times 0 = -6$; the second is $0 \times 3 + 1 \times 0 = 0$.
2. One scalar, -2 , scales both coordinates. Its negative sign reverses orientation; its absolute value 2 doubles length.

004-5 / TRANSFER

Let $P = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$ and $v = (2, 0)$. P sends v to the origin. Does v count as an eigenvector? Would the zero vector itself count?

Yes: $Pv = (0, 0) = 0v$, so v is an eigenvector with $\lambda = 0$. The zero vector is not an eigenvector.

1. The input $(2, 0)$ is nonzero. Multiplication gives $(0 \times 2 + 0 \times 0, 0 \times 2 + 1 \times 0) = (0, 0)$.
2. The scalar 0 works: $0(2, 0) = (0, 0)$. The output has no direction, but the eigenvector definition still holds. The zero input is excluded because it would satisfy $A0 = \lambda 0$ for every λ .

004-6 / TRANSFER

$R = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$ rotates every plane arrow 90° counterclockwise. Test $(1, 0)$, then decide whether any nonzero real vector can be an eigenvector of R .

$R(1, 0) = (0, 1)$, not a multiple of $(1, 0)$. No nonzero real vector is an eigenvector of this 90° rotation.

1. $R(1, 0) = (0 \times 1 + (-1) \times 0, 1 \times 1 + 0 \times 0) = (0, 1)$. The input lies on the horizontal line and the output on the vertical line.
2. The same quarter-turn moves every nonzero real arrow off its line, while preserving nonzero length. Real scaling cannot make that turn, and $\lambda = 0$ cannot describe a nonzero output. Thus no real λ and nonzero real v satisfy $Rv = \lambda v$.