

One Number Acting on an Entire Matrix

Read the object outside the brackets before you choose an operation.

THE QUESTION

If we put 3 beside a grid of numbers, which entries does it multiply?

Start with $M = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, a two-row, two-column grid of numbers called a matrix. The 3 beside M is one number, called a scalar. We want to calculate $3M$ and see where each answer goes.

Name the objects

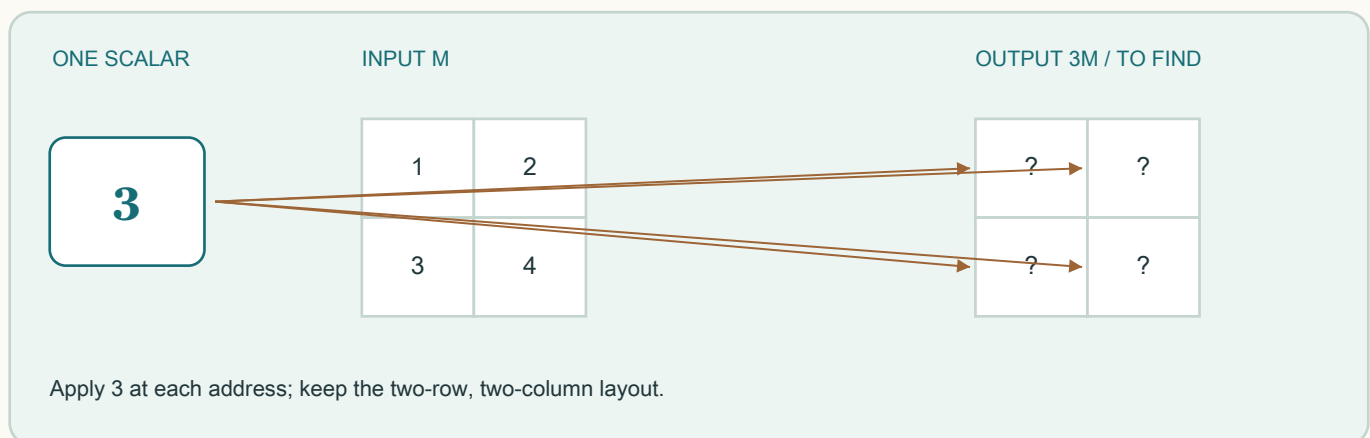
3 - One scalar: a single number that will multiply each entry of M.

M - A whole matrix, here arranged in two rows and two columns. M is the object being scaled; it is not one of its entries.

1, 2, 3, 4 - The four entries of M. Each has an address: row 1 contains 1 and 2; row 2 contains 3 and 4.

$3M$ - Scalar multiplication: one number applied to a whole matrix. This differs from multiplying two matrices together.

One value, four destinations



The single scalar reaches each of the four entries. Every answer keeps its original row and column address.

Work it through

Find $3M$ for $M = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$. Keep the entries in their original positions as you calculate.

1. Name the objects

$$3 \times M, \text{ with } M = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

The 3 is one scalar. M is one 2×2 matrix. Because the left object is a number, there is no row-column pairing to choose.

2. Keep each address

$$3M = \begin{bmatrix} 3 \times 1 & 3 \times 2 \\ 3 \times 3 & 3 \times 4 \end{bmatrix}$$

Copy the two-row, two-column layout. At every address, multiply the entry already there by 3. For example, the row 2, column 1 entry becomes 3×3 .

3. Finish the arithmetic

$$3M = \begin{bmatrix} 3 & 6 \\ 9 & 12 \end{bmatrix}$$

Four input entries give four output entries. Scalar multiplication changes their values, not their locations or the shape of the matrix.

$$3 \times \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 3 & 6 \\ 9 & 12 \end{bmatrix}.$$

The compact rule

$$3M = 3 \times \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

Three multiplies the whole matrix M. Multiply each of its four entries by 3, keeping every answer at the same row and column address.

If the scalar has a name

The same move works with a named scalar. If a rule named k assigns a number to each x , $k(x)$ means the number assigned to the x we choose. Let a , b , c , and d stand for the four entries of M ; m_{i_j} means its entry in row i , column j . Once x is chosen, use the single value $k(x)$ at every address.

$k(x) \begin{bmatrix} a, b \\ c, d \end{bmatrix} = \begin{bmatrix} k(x)a, k(x)b \\ k(x)c, k(x)d \end{bmatrix}$. In entry notation, $(k(x)M)_{ij} = k(x)m_{ij}$.

A tempting wrong turn

Multiply the number by just the first entry, or treat $3M$ as if two matrices need a row-column rule.

The brackets describe one matrix, and 3 is a scalar, not another matrix. Every entry receives the scalar once. If the expression were MN with both M and N matrices, it would be a different instruction with a shape check and matched row-column products. An arithmetic slip, such as writing $3 \times 4 = 11$, is different from misreading which operation the expression requests.

Where it appears again

A single factor often scales an entire table of numbers. Later, $Av = \lambda v$ says that one scalar λ multiplies every component of a vector v ; the left side, Av , is a matrix-vector product, while the right side, λv , is scalar multiplication. A letter's role comes from the expression and its stated context. In complex-number work, i may be the imaginary unit ($i^2 = -1$), so iM would scale each entry by i if complex entries are allowed. In m_{i_j} , the subscript i is a row index instead: it selects an address and is not a number placed outside the matrix. Do not decide from the letter alone.

One-minute reference

- One scalar times one matrix: multiply every entry by that scalar and keep the matrix's shape.
- For $M = [m_{i_j}]$ and scalar s , the entry at address (i, j) in sM is $s \times m_{i_j}$.
- If $s = k(x)$, choose x and evaluate $k(x)$ before using the result at every entry.
- Two matrices written together call for the matrix-product rule, which is a different operation.
- A symbol's role depends on position and context: i can be an index in m_{i_j} or the imaginary unit in complex-number work.

Try it yourself

Use the hints if you need a starting move. The worked answers are in the separate answer key.

003-1 / RECOGNIZE

P and Q are matrices. Which expression asks for scalar multiplication: $5P$ or PQ ? What clue settles it?

Hint: Name the object to the left of P in each expression.

003-2 / RECOGNIZE

M has 2 rows and 3 columns, and s is a scalar. What is the shape of sM ? If the entry of M at row 2, column 3 is m_{23} , what is the entry at that address in sM ?

Hint: The scalar changes each value. Does it add or remove any addresses?

003-3 / APPLY

Calculate $-2 \times \begin{bmatrix} 3 & -1 \\ 0 & 4 \end{bmatrix}$.

Hint: Write -2 times each of the four entries before simplifying.

003-4 / APPLY

Calculate $(1/2) \times \begin{bmatrix} 6 & 0 \\ -4 & 10 \end{bmatrix}$.

Hint: Halve every entry, including zero and the negative entry.

003-5 / TRANSFER

Let $k(t) = 2t - 1$ and $P = \begin{bmatrix} 1 & 0 \\ 2 & -3 \end{bmatrix}$. Find $k(2)P$. What would $k(t)P$ mean before a value of t is chosen?

Hint: Evaluate $k(2)$ first. For the symbolic part, leave $k(t)$ as one factor at each address.

003-6 / TRANSFER

In complex-number work, $i^2 = -1$. Compare iP with p_{i_j} , where $P = [p_{i_j}]$. What job does i do in each expression? If $P = \begin{bmatrix} 1 & 2 \end{bmatrix}$, what is iP ?

Hint: Look for the position of i: outside the brackets or down in an entry's address.

Further reading

Georgia Tech, Interactive Linear Algebra, §3.4 Matrix Multiplication: <https://textbooks.math.gatech.edu/ila/matrix-multiplication.html>

Georgia Tech, Interactive Linear Algebra, Appendix A Complex Numbers: <https://textbooks.math.gatech.edu/ila/complex-numbers.html>