

Worked answer key / 003

One Number Acting on an Entire Matrix

Each answer explains the reasoning as well as the result. The numbered prompts match the lesson and card deck.

003-1 / RECOGNIZE

P and Q are matrices. Which expression asks for scalar multiplication: $5P$ or PQ ? What clue settles it?

$5P$ asks for scalar multiplication; PQ asks for matrix multiplication.

1. In $5P$, 5 is one number, so it multiplies every entry of P .
2. In PQ , both named objects are matrices. Its entries require the matrix-product rule, assuming their shapes are compatible.

003-2 / RECOGNIZE

M has 2 rows and 3 columns, and s is a scalar. What is the shape of sM ? If the entry of M at row 2, column 3 is m_{23} , what is the entry at that address in sM ?

sM is 2×3 , and its row 2, column 3 entry is $s \times m_{23}$.

1. M has six addresses arranged in 2 rows and 3 columns; scaling leaves those addresses in place.
2. At each address, multiply the existing entry by s . At $(2, 3)$, that gives $s \times m_{23}$.

003-3 / APPLY

Calculate $-2 \times \begin{bmatrix} 3 & -1 \\ 0 & 4 \end{bmatrix}$.

$\begin{bmatrix} -6 & 2 \\ 0 & -8 \end{bmatrix}$

1. Keep the two rows: $[-2 \times 3, -2 \times (-1)], [-2 \times 0, -2 \times 4]$.
2. The products are $-6, 2, 0$, and -8 in their original positions, giving $\begin{bmatrix} -6 & 2 \\ 0 & -8 \end{bmatrix}$.

003-4 / APPLY

Calculate $(1/2) \times \begin{bmatrix} 6 & 0 \\ -4 & 10 \end{bmatrix}$.

$\begin{bmatrix} 3 & 0 \\ -2 & 5 \end{bmatrix}$

1. At each address, multiply by $1/2$: $[6/2, 0/2], [-4/2, 10/2]$.
2. Simplify without moving entries: $\begin{bmatrix} 3 & 0 \\ -2 & 5 \end{bmatrix}$.

003-5 / TRANSFER

Let $k(t) = 2t - 1$ and $P = \begin{bmatrix} 1 & 0 \\ 2 & -3 \end{bmatrix}$. Find $k(2)P$. What would $k(t)P$ mean before a value of t is chosen?

$k(2)P = \begin{bmatrix} 3 & 0 \\ 6 & -9 \end{bmatrix}$. Before choosing t , $k(t)P = \begin{bmatrix} k(t) & 0 \\ 2k(t) & -3k(t) \end{bmatrix}$.

1. $k(2) = 2 \times 2 - 1 = 3$, so the instruction becomes $3P$.
2. Multiply each entry of P by 3: $[3 \times 1, 3 \times 0], [3 \times 2, 3 \times (-3)] = \begin{bmatrix} 3 & 0 \\ 6 & -9 \end{bmatrix}$.
3. With t unspecified, use the same factor $k(t)$ at every address: $\begin{bmatrix} k(t) & 0 \\ 2k(t) & -3k(t) \end{bmatrix}$.

003-6 / TRANSFER

In complex-number work, $i^2 = -1$. Compare iP with p_{i_j} , where $P = [p_{i_j}]$. What job does i do in each expression? If $P = \begin{bmatrix} 1 & 2 \end{bmatrix}$, what is iP ?

In iP , i is the imaginary-unit scalar, so $iP = \begin{bmatrix} i & 2i \end{bmatrix}$. In p_{i_j} , i is the row index, not a multiplier.

1. The statement $i^2 = -1$ declares i to be the imaginary unit in this context. Placed before P , it scales each entry; this gives $[i \times 1, i \times 2] = [i, 2i]$. The output has complex entries.
2. In p_{i_j} , subscripts identify an entry at row i and column j . That i names a position. The same printed letter can have a different role in a different context.