

Worked answer key / 002

Rows, Columns, and Why You Don't Multiply Everything

Each answer explains the reasoning as well as the result. The numbered prompts match the lesson and card deck.

002-1 / RECOGNIZE

In $A = \begin{bmatrix} 2 & 1 & 3 \\ 4 & 2 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 \\ 3 & 1 \\ 2 & 4 \end{bmatrix}$, find a_{23} and b_{32} . Say each address as row, column before reading its value.

$a_{23} = 1$ and $b_{32} = 4$.

1. A's second row is (4, 2, 1); its third entry is 1.
2. B's third row is (2, 4); its second entry is 4. An address names one entry, not the whole matrix.

002-2 / RECOGNIZE

Let $A = \begin{bmatrix} 2 & 1 & 3 \\ 4 & 2 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 2 \\ 3 & 1 \\ 2 & 4 \end{bmatrix}$, and $C = AB$. For c_{12} , which row of A and column of B must meet? How many paired products make that one entry?

Use A row 1, (2, 1, 3), with B column 2, (2, 1, 4). That entry has three paired products.

1. The first subscript 1 selects A row 1; the second subscript 2 selects B column 2.
2. Their positions match three times: 2 with 2, 1 with 1, and 3 with 4. The shape's shared 3 sets the count.

002-3 / APPLY

Let $A = \begin{bmatrix} 2 & 1 & 3 \\ 4 & 2 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 2 \\ 3 & 1 \\ 2 & 4 \end{bmatrix}$, and $C = AB$. Calculate c_{22} . Write the chosen row and column before the arithmetic.

$c_{22} = 4 \times 2 + 2 \times 1 + 1 \times 4 = 8 + 2 + 4 = 14$.

1. Address 22 fixes row 2 of A and column 2 of B.
2. The corresponding pairs are 4 with 2, 2 with 1, and 1 with 4. Their products add to 14.
3. Place 14 at row 2, column 2 of C.

002-4 / APPLY

Let $A = \begin{bmatrix} 2 & 1 & 3 \\ 4 & 2 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 \\ 3 & 1 \\ 2 & 4 \end{bmatrix}$.

Is BA defined? If so, give its shape and top-left entry.

BA has shape 3×3 . Its top-left entry is $1 \times 2 + 2 \times 4 = 10$.

1. The adjacent dimensions in $(3 \times 2)(2 \times 3)$ match at 2; the outside dimensions are 3 and 3.
2. B row 1 is (1, 2), and A column 1 is (2, 4). Pair their positions and add $2 + 8 = 10$.

Worked answers / continued

002-5 / APPLY

Let $P = \begin{bmatrix} 1 & 0 \\ 2 & 3 \end{bmatrix}$ and $Q = \begin{bmatrix} 4 \\ 5 \end{bmatrix}$. Find PQ and its shape.

$PQ = \begin{bmatrix} 4 \\ 23 \end{bmatrix}$, a 2×1 matrix.

1. The shapes $(2 \times 2)(2 \times 1)$ fit, and the outside dimensions give 2×1 .
2. The first entry is $1 \times 4 + 0 \times 5 = 4$. The second is $2 \times 4 + 3 \times 5 = 8 + 15 = 23$.
3. Put the two entries in one column: $\begin{bmatrix} 4 \\ 23 \end{bmatrix}$.

002-6 / TRANSFER

A is 2×3 , B is 3×2 , and D is any 2×3 matrix. Which of AB, BA, AD, and DA are defined? Give each valid output shape.

AB is defined and is 2×2 . BA is defined and is 3×3 . AD and DA are undefined.

1. AB has shapes $(2 \times 3)(3 \times 2)$: the adjacent 3s match, giving 2×2 .
2. BA has shapes $(3 \times 2)(2 \times 3)$: the adjacent 2s match, giving 3×3 .
3. AD and DA both have shapes $(2 \times 3)(2 \times 3)$. Their adjacent dimensions are 3 and 2, so neither product is defined.

002-7 / TRANSFER

Without calculating entries, describe $(4 \times 9)(9 \times 6)$: output shape, total output entries, and paired products per entry.

The output is 4×6 , with 24 entries and 9 paired products in each entry.

1. The adjacent 9s match, so the product is defined. The outside dimensions 4 and 6 give its shape.
2. A 4×6 matrix has $4 \times 6 = 24$ entries. The shared 9 means each entry adds nine corresponding products.

002-8 / TRANSFER

Let $A = \begin{bmatrix} 2 & 1 & 3 \\ 4 & 2 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 2 \\ 3 & 1 \\ 2 & 4 \end{bmatrix}$, and $C = AB$. Someone tries nine every-to-every products for c_{21} . Which row and column should they choose, how should they pair positions instead, and what is c_{21} ?

Make three corresponding products: $4 \times 1 + 2 \times 3 + 1 \times 2 = 12$.

1. For c_{21} , A row 2 is (4, 2, 1) and B column 1 is (1, 3, 2).
2. Pair first with first, second with second, and third with third, giving 4, 6, and 2.
3. Add those three products to get 12. The shared dimension is 3, so nine every-to-every products answer a different question.