

What Is This Formula Asking Me to Do?

A field guide to reading indices, sums, and one instruction at a time.

THE QUESTION

Given two number grids, how do we find one chosen number in their product?

Start with $A = \begin{bmatrix} 2 & 1 & 3 \\ 4 & 2 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 \\ 3 & 1 \\ 2 & 4 \end{bmatrix}$. These grids of numbers are matrices. Their product is another matrix, $C = AB$. Find its entry in row 2, column 1 before reading the compact rule for that calculation.

Name the objects

A, B, C - Whole matrices: grids of numbers. Here C is the matrix product AB.

Hold the destination still. Move through the matching positions.

A / 2 rows x 3 columns

2	1	3
4	2	1

A row 2: (4, 2, 1)

Match positions: 4 with 1, 2 with 3, 1 with 2.

B / 3 rows x 2 columns

1	2
3	1
2	4

B column 1: (1, 3, 2)

C = AB / 2 x 2

?	?
?	?

c₂₁: still unknown

Work it through

Let $A = \begin{bmatrix} 2 & 1 & 3 \\ 4 & 2 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 \\ 3 & 1 \\ 2 & 4 \end{bmatrix}$. Find the number in row 2, column 1 of $C = AB$.

1. Name the address

c_{21} = the entry in C at row 2, column 1

The first subscript gives the row and the second gives the column. This address asks for one number, not the whole matrix C.

2. Choose the strips

A row 2 = (4, 2, 1); B column 1 = (1, 3, 2)

The first number in each strip is a pair, then the second numbers, then the third numbers.

3. Multiply matching positions

$4 \times 1, 2 \times 3, 1 \times 2$

Position 1 meets position 1, and so on. The three pairs give three products, rather than every possible combination.

4. Add the products

$c_{21} = 4 \times 1 + 2 \times 3 + 1 \times 2 = 4 + 6 + 2 = 12$

This gives one entry, not the whole matrix C.

$c_{21} = 12$

The compact rule

$$c_{ij} = \sum_{z=1}^3 a_{iz} b_{zj}$$

To find the entry of C in row i and column j, let z run through 1, 2, and 3. At each position z, multiply the entry in row i of A by the matching entry in column j of B. Add those three products.

Read the symbols in that rule

a_{iz} , b_{zj} , c_{ij} - Individual entries. The first subscript gives a row and the second gives a column. For example, c_{21} is one number in row 2, column 1 of C.

i, j - The fixed address of the one output entry we are finding: row i and column j.

z - The running index. It changes from 1 to 3 and links corresponding positions in A's row and B's column.

Σ - A request to add. The lower and upper labels set the first and last values of the running index. The sum covers the complete product $a_{iz} b_{zj}$.

= - Equality: the expression on the right gives the value of the entry on the left. In a definition, some authors use $:=$ to flag that they are introducing a name.

A sum without matrices

$$\Sigma_{k=1}^4 2k$$

Let k take the values 1, 2, 3, and 4. Double each value, then add the four results.

$$2(1) + 2(2) + 2(3) + 2(4) = 2 + 4 + 6 + 8 = 20$$

The letter k is a placeholder for the changing value. Calling it t instead would leave the answer at 20. The 2k means 2 times k, not k squared.

A tempting wrong turn

Let i, j, and z all change together, or multiply every number in the chosen row by every number in the chosen column.

For one c_{ij} , i and j name a single destination and stay fixed. Only z runs through the sum. Three matching positions make three products, not nine. A later destination can use different i and j.

Conventions depend on context

Below or above: A subscript often identifies an entry, as in a_{21} ; an exponent often means a power, as in $x^2 = x \times x$. These are conventions, not a universal visual code. A superscript can also be a component label in another subject, so check the definitions nearby.

Capital or lowercase: Here A names a whole matrix and a_{21} names one of its entries. That is a common convention, but an author can choose different letters. Read the local definition before relying on case alone.

Equal or defined as: An equals sign says the two sides have the same value. A colon-equals sign ($:=$) sometimes announces a definition. In context, an ordinary equals sign may also introduce a definition. Neither sign means "perform these steps in this order" by itself.

Where it appears again

In the next note, Rows, Columns, and Why You Don't Multiply Everything, this reading rule becomes a complete matrix product. Later, the same habit - identify the object, hold some indices fixed, and let another index vary - helps with matrix transformations and repeated application.

One-minute reference

- Name the output first: c_{ij} is one entry, while C is the whole matrix.
- Subscript ij gives the output address: row i, column j.
- In one sum, i and j stay fixed; z takes every integer from 1 through 3.
- At each z, multiply a_{iz} by b_{zj} . Add the complete products.
- Check whether a raised mark is a power or a label from its local definition.

Try it yourself

Use the hints if you need a starting move. The worked answers are in the separate answer key.

001-1 / RECOGNIZE

In a grid P, which address does p₄₂ name?

Hint: Read the first subscript as the row and the second as the column.

001-2 / RECOGNIZE

Let A be 2×3 , B be 3×2 , and $C = AB$. Its entry rule is $c_{ij} = \sum_{z=1}^3 a_{iz} b_{zj}$. While finding c_{21} , which symbols stay fixed and which one runs?

Hint: Start with the requested output address 21. Then look at the letter below Σ .

001-3 / APPLY

Expand and evaluate $\sum_{k=1}^4 (k + 1)$.

Hint: Substitute $k = 1$, then 2, then 3, then 4 into the entire expression $(k + 1)$.

001-4 / APPLY

With $A = \begin{bmatrix} 2 & 1 & 3 \\ 4 & 2 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 \\ 3 & 1 \\ 2 & 4 \end{bmatrix}$, expand and calculate c_{12} in $C = AB$.

Hint: The address 12 selects row 1 of A and column 2 of B. Pair their first, second, and third positions.

001-5 / TRANSFER

Expand and evaluate $\sum_{r=2}^4 3r$. Does changing the running letter from r to t change the value?

Hint: The first value is 2, not 1. Visit $r = 2, 3$, and 4.

001-6 / TRANSFER

A rule says $d_{ij} = \sum_{q=1}^5 x_{iq} y_{qj}$. How many products make one entry d_{ij} , and does the 5 say how many entries D has?

Hint: Count the values of q while holding one output address ij fixed.

Further reading

Interactive Linear Algebra, §3.4 Matrix Multiplication - Georgia Tech: <https://textbooks.math.gatech.edu/ila/matrix-multiplication.html>